International Journal of Theoretical Physics, Vol. 31, No. 9, 1992

Quantum Logics Seen As Quantum Testability Theories

Claudio Garola[1]

Received February 10, 1992

We confute logical relativism and forward an alternative epistemological thesis according to which nonstandard "truth-theories" are considered theories of some metalinguistic concepts which do not coincide with truth, this latter concept being exhaustively described by Tarski's truth theory. We illustrate our viewpoint by showing that quantum logics can be interpreted as quantum physical theories of the metalinguistic concept of testability in the framework of a suitable classical language (with Tarskian semantics).

1. INTRODUCTION

Many nonclassical logics have been constructed since Brouwer's proposal of intuitionistic logic. Presently, we have nonclassical two-valued logics, three-valued logics, infinite-valued logics. Among these, quantum logic (QL) has a privileged role, since it is suspected to be the basic logical apparatus underlying quantum physics (QP).

Every nonstandard logic can be supposed to subtend a nonclassical truth theory, so that we have nonclassical two-valued truth theories, many-valued truth theories, fuzzy truth theories, and so on. This proliferation of truth theories strongly favors philosophical relativism regarding the concept of truth; yet, it has been considered by many authors [in particular, by Putnam (1979)] as an achievement comparable with the discovery of non-Euclidean geometries.

I do not agree with this viewpoint; indeed, I think that logical theories and specific theories (like geometries) have different epistemological statuses, and that the belief in the relativity of logic is exposed to many relevant philosophical and epistemological objections.

[1]Dipartimento di Fisica dell'Universita' di Lecce, Italy.

0020-7748/92/0900-1639$06.50/0

I would like to discuss briefly these objections in the first part of this paper and to present an epistemological alternative. According to this, every nonclassical "truth theory" actually is a theory of some metalinguistic concept which is different from the concept of truth, while the properties of truth are exhaustively described by the Tarski truth theory, hence by classical logic (CL).

This viewpoint can be supported by two examples at least. First, it has recently been proved by Dalla Pozza (1991) that the intuitionistic theory of truth can be interpreted as a theory of the pragmatic concept of justification. Second, the "truth theory" underlying QL can actually be interpreted as a theory of the pragmatic concept of testability in QP (which is different from the classical concept of truth) by making use of the results which I have obtained in a recent paper (Garola, 1991).

QL, of course, is the main topic in this conference. Thus, I will try to illustrate the latter example in the second part of this paper by showing that the formulas of QL can be interpreted as subsets of testable formulas of a classical language with Tarskian semantics and that the structure properties of QL can be interpreted as properties of the concept of testability in QP.

For brevity's sake I will make considerable use of intuitive arguments and short-cuts, and only sketch the scheme which can be followed when a rigorous treatment of the subject is desired; each step of the scheme can actually be made with the aid of the paper quoted above.

As an immediate consequence of my discussion, it follows that a nonclassical logic is not strictly needed in QP, which agrees with the opinion of many theorists (in particular, those belonging to the Geneva school), but is opposite to the beliefs of other theorists and logicians (see, for instance, Jammer, 1974; Holdsworth and Hooker, 1983).

2. A NEW EPISTEMOLOGICAL PERSPECTIVE

I would like to underline first some basic notions about formal languages and their interpretations in order to make my arguments and my thesis clearer.

The noun "formal language" is usually intended to denote a pair $L = (\mathscr{A}, \Psi)$, with $\mathscr{A}$ and *alphabet*, i.e., a set of symbols classified according to syntactic categories (e.g., *descriptive*, *logical*, *auxiliary* symbols) and Ψ a set of *well-formed formulas* (briefly, wffs) constructed by means of the symbols in $\mathscr{A}$ and of suitable *formation rules*.

Whenever a formal language L has been constructed, the problem occurs of endowing it with an *effective semantical interpretation*. A first step in this direction can be made by providing a *formal semantical interpretation* for L, i.e., a triple $\mathscr{M} = (\mathscr{D}, \rho, \mathscr{T})$, where $\mathscr{D}$ is an abstract set called *domain*, or *universe*, ρ is an *assignment function* which makes every primitive descriptive symbol of L correspond to a suitable entity in $\mathscr{D}$ (precisely, ρ maps every individual constant of L on an element in $\mathscr{D}$ and every n-adic predicate of L onto a suitable subset of $\mathscr{D}^n$), and $\mathscr{T}$ is a *truth theory* where the concepts of *truth*, *validity* (or *logical truth*), and *logical consequence* are (recursively) defined. The formal interpretation $\mathscr{M}$ yields a complete interpretation of the logical symbols in L, defines exhaustively the concept of logical truth, and represents a regulative criterion to which any effective semantical interpretation must conform (Thomason, 1974).

If a formal semantical interpretation is provided, an effective interpretation can be obtained by suitably mapping a fragment F of the natural language on L; then the formal language L with the formal semantical interpretation $\mathscr{M}$ is said to be *adequate* to F whenever such a mapping exists (it is obviously required that the semantics of F be compatible with the formal semantics defined in L).

It is well known that the truth theory in the above procedure can be substituted, under the assumption that some completeness requirements are satisfied, by a suitable formal calculus $\mathscr{C}$, i.e., by a pair (A, R), where A is a set of logically true formulas of L, called *logical axioms*, and R is a set of *primitive inference rules*. Whenever the theory $\mathscr{T}$ is given, the formal semantical interpretation uniquely determines (up to logical equivalence) the calculus $\mathscr{C}$ and, conversely, whenever the calculus $\mathscr{C}$ is assigned in place of $\mathscr{T}$, a truth theory is induced in such a way that $\mathscr{M}$ is a model for $\mathscr{C}$.

Let us now consider the existing formal semantical interpretations. Of course a standard truth theory, hence a standard logical calculus, exists: the Tarskian truth theory, which subtends the formal calculus of classical logic. Yet, many truth theories and calculi have been proposed which are not logically equivalent to the classical ones, thus yielding nonstandard formal semantical interpretations. Furthermore, it has been proved that effective interpretations actually exist of languages endowed with these nonclassical formal interpretations. Thus, the widespread opinion arises that the concept of truth may change with the fragment of the natural language, the classical concept being only one of the possible ones.

Besides, one can reason as follows. It is well known that the set of axioms and inference rules of $\mathscr{C}$, or the truth theory in $\mathscr{M}$, define a poset $\mathscr{L}$ (usually, a lattice, called the *algebra of the propositions* of L) whose elements are classes of logically equivalent wffs of L. Whenever a fragment F of the natural language is mapped onto L, this structure could be said to formalize properties of the concept of truth in F; hence, if L bears a nonstandard formal semantical interpretation, $\mathscr{L}$ illustrates the differences between the concept of truth underlying F and the classical concept (whose properties are formalized by a Boolean algebra of propositions).

The above point of view, which strongly supports the thesis of the "locality of logic" that has been defended by many authors (e.g., Dalla Chiara, 1974), is exposed to some relevant philosophical objections. First, it implies that the logical apparatus which underlies a given theory may depend on the theory itself, since the latter selects the fragment F of the natural language by means of which it is expressed; thus, we have no "rationality principle" which allows a preliminary choice between theories on the basis of external criteria like "consistency" of "coherence," and any theory can, in principle, justify itself. Second, it involves the fact that no rule for selecting admissible languages and calculi can be given, since for any formal language L endowed with a nonstandard formal semantical interpretation it cannot be excluded in principle that a fragment F of the natural language can be found such that L is adequate to F. In addition, the thesis of locality of logic often leads to identifying different truth modes, like the truth modes of logical and physical laws, since it is naturally linked to the thesis that "logic is empirical" (e.g., Putnam, 1969; Finkelstein, 1979*a*,*b*; Drieschner, 1977), though it does not coincide with it; furthermore, it may also engender some nontrivial problems of compatibility whenever overlapping fragments of the natural language are mapped onto inequivalent languages.

However, the thesis that different concepts of truth underlie different theories only constitutes a possible viewpoint. I intend to forward here the following alternative thesis which seems to overcome the above philosophical objections: *the different algebraic structures that are induced by nonclassical formal semantical interpretations formalize properties of metalinguistic concepts that do not coincide with the concept of truth whose formal properties are exhaustively expressed by Tarski's truth theory.* If this thesis is accepted, every non-Tarskian "truth theory" will be considered as a theory regarding a metalinguistic concept, different from truth (like the concept of testability, whose relevance in physics will be discussed in the sequel) in the framework of a classical language (possibly extended).

This epistemological position finds its ancestors in Carnap (1949, 1966), Popper (1969) and in the principle by Quine (1970), "change of logic, change of subject." It entails a rather radical change in perspectives with respect to some current lines of thought in nonstandard logics and its consequences are far-reaching. For instance, it implies that different algebraic structures can coexist in the same theory, so that, in particular, the logical apparatus of a given theory does not necessarily depend on the theory itself; indeed the nonstandard structures induced by the theory can be interpreted as expressing formal properties, inside the theory, of some specific metalinguistic concept, while the formal properties of the truth concept remain unaltered. Furthermore, our viewpoint allows a classification of nonstandard languages, which can be classified according to the metalinguistic concept formalized by means of their algebraic structure. Finally, it is apparent that the analysis of the truth modes of different kinds of laws is not affected by the existence of distinct algebraic structures which formalize the properties of concepts which are different from the concept of truth.

It is also interesting to note that the above thesis induces a particularly interesting suggestion for our purposes in the present paper. Let us consider some distinct formal languages, say $L_1, L_2, \ldots, L_n$, endowed with nonstandard formal semantical interpretations which are not requested to be logically equivalent, and let $F_1, F_2, \ldots, F_n$ be fragments of the natural language that can be regimented onto $L_1, L_2, \ldots, L_n$, respectively. One can imagine that a formal language L can be constructed, endowed with a standard formal semantical interpretation, onto which a fragment F of the natural language can be regimented which embodies $F_1, F_2, \ldots, F_n$. Then, the algebraic structure of L is classical, the fragments $F_1, F_2, \ldots, F_n$ can be considered as obtained by choosing in F sets of statements possessing some suitable metalinguistic property (testability, provability, etc.) which can be different for different fragments, and the algebraic structure on any fragment (which might have nothing in common with that of L) can be regarded as formalizing properties of the metalinguistic concept used in selecting the fragment itself.

Let us come now to a second epistemological thesis, which is secondary to the previous one, but relevant for our purposes. More precisely, if we accept the above perspectives, we are naturally led to look for metalinguistic concepts which are philosophically prominent and can play a role in determining nonclassical structures within the framework of classical languages. Whenever we direct our attention at physics, the concept of testability (also, epistemic accessibility) immediately seems to be a suitable candidate: it suffices to think that QP has been constructed since the very beginning by identifying the semantical concept of meaning and the pragmatic concept of testability, and rejecting as "physically meaningless" every nontestable statement.

Thus, we claim that *the concept of testability plays a fundamental role in every physical theory, since every physical theory embodies a theory of testability (possibly implicit) which selects the set of all the interpreted formulas (or "statements") considered to be testable (at least in principle) in the set of all the formulas belonging to the language of the theory. Consequently, the physical laws are accepted or rejected according to their success in predicting the truth values of testable statements, while the prediction of the truth values of nontestable statements is usually considered irrelevant by physicists.*

It should be noted that a number of important consequences occur whenever this perspective is adopted. First, truth and testability do not necessarily coincide (this will be confirmed by the particular case of QL), though they may coincide in special theories, so that we are led to reject the early neopositivistic verification theory of meaning. Second, we obtain that, in every physical theory, the set of statements which are testable is a subset (usually proper) of the set of all formulas which are endowed with a truth value. Third, we expect that the properties of the concept of testability are theory-dependent (while those of the concept of truth should be theory-independent according to our viewpoint). Fourth, the classical problem of the completeness of a physical theory can be reformulated by saying that the theory is *t-complete* if it determines the truth values of all the interpreted wffs that the theory itself classifies as testable, while it is *s-complete* if it determines the truth values of all the interpreted wffs in the language of the theory; it is then apparent that s-completeness (which coincides with the standard notion of completeness) is not relevant from a physicist's point of view.

3. CLASSICAL FOUNDATIONS OF QUANTUM LOGICS

Let us now consider QL. In this case it is usually assumed that the formal language, whatever it may be, must be endowed with a nonclassical formal semantical interpretation (or calculus) whose algebra of propositions has the properties suggested by a number of existing axiomatized or semi-axiomatized approaches to the foundations of QP, i.e., it is a complete, orthocomplemented, weakly modular, atomic lattice which also satisfies the covering law. Thus, it seems obvious to assert that a quantum concept of truth exists which is different from the classical one, which is retained to underlie classical physics (CP). Yet this position is exposed to the objections debated before. In addition, some effective semantical interpretations resulting from this assumption seem to be anomalous if compared with the effective interpretation provided by foundational physicists for their algebraic structures.

By applying our previous arguments in particular to QL, we see that these difficulties do not occur if one accepts the alternative point of view introduced in this paper: in this case, the algebraic structure of QL is thought of as formalizing properties of a metalinguistic concept which does not coincide with the concept of truth. Moreover, we are naturally led to guess that the concept of *testability* or some suitable refinement of it is the required concept.

I want to discuss here whether this conjecture is true; to be precise, I intend to show that the algebraic structure of QL can be obtained as an empirical structure which formalizes properties of the pragmatic concept of testability in QP in the framework of a suitable classical language.

In order to obtain this result, a complete treatment could be carried out according to the following scheme.

(a) Construction of a formal language L, suitably extended by means of new operators and/or quantifiers.
(b) Assignment of a formal semantical interpretation $\mathcal{M}$ of L, which constitutes a model-theoretic semantics for L, embodying a classical truth theory.
(c) Assignment of an effective semantical interpretation of L which conforms to the regulative criteria established by the formal semantical interpretation and makes L suitable for expressing the basic concepts and laws of QP.
(d) Statement of the basic laws of QP by means of formulas (or schemes of formulas) of L.
(e) Construction of one (or more) formal language L_q endowed with an effective interpretation which induces the algebraic structure of QL on the set Ψ_q of all the wffs of L_q (hence it subtends a nonclassical "truth theory").
(f) Translation τ of L_q into L which maps the set Ψ_q into a subset $\tau(\Psi_q)$ of wffs of L, in such a way that the effective interpretation of L_q is preserved (the translation will be weak, in the sense that the interpretation of the logical signs is not necessarily preserved).
(g) Proof that the mathematical structure induced by the laws of QP on $\tau(\Psi_q)$ can be identified (up to suitable homomorphisms) with the quantum logical structure of Ψ_q.
(h) Proof (via the effective interpretation) that the metalinguistic concept of testability (or some appropriate refinement of it) actually characterizes the subset $\tau(\Psi_q)$.
(i) Interpretation of the structure properties of Ψ_q as properties in QP of the (refined) concept of testability.

This program has actually been realized in the paper quoted above (Garola, 1991), apart from some minor changes. However, this approach is exceedingly complicated because it aims to be self-consistent and wants to treat a number of topics during the preparation of the basic tools for translating "quantum logics" into L. Indeed, (i) L is enlarged by introducing a new family of quantifiers (the *statistical* quantifiers) so that statements on conditional frequencies can be suitably formalized in L; (ii) the concept of physical laboratory is formalized and used in order to characterize the truth modes of different classes of laws (logical, analytical, physical) in our framework; (iii) a distinction between probabilistic and frequential statements is introduced, and a general principle is stated which allows us to translate a probabilistic statement into a frequential statement with a known degree of approximation; (iv) some relevant physical assumptions are made explicit that are implicit both in CP and in QP; (v) new characterizations of *pure states* and *fuzzy properties* are obtained; (vi) many-valued QL is recovered, together with two-valued QL, in a classical framework; (vii) CP is suitably characterized and the difference between CP and QP is discussed; and (viii) some hints are given for solving classical "paradoxes" in QP.

If these topics are ignored and some plausible additional assumptions are accepted, we can restrict ourselves to a simplified classical language L_c, and the arguments in the aforesaid approach can be greatly simplified and schematized, even if we lose generality and conceptual rigor.

I will briefly deal with this simplified treatment here in order to give an intuitive insight into the problem.

4. A CLASSICAL LANGUAGE FOR QUANTUM LOGIC

Let us concretely build up the simplified classical language L_c. We assume that L_c is a language of a classical first-order predicate logic, with alphabet $\mathscr{A}_c$ and set of well-formed formulas (wffs) Ψ_c. The alphabet $\mathscr{A}_c$ contains the following descriptive signs.

(a) Individual variables: $x, y, \ldots$.
(b) Monadic predicative constants, divided into two classes: (i) symbols of state: $S, S_1, S_2, \ldots$; (ii) symbols of exact effect: $0, 1, E, E_1, E_2, \ldots$.

Furthermore, $\mathscr{A}_c$ contains the standard connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ and quantifiers $\ni$, $\forall$ of CL. Finally, the auxiliary signs in $\mathscr{A}_c$ are limited to round parentheses.

Then, Ψ_c is obtained by means of the signs in $\mathscr{A}_c$ and of the standard formation rules of the classical predicate logic. In addition, we denote the sets of (individual) variables, symbols of state, and symbols of exact effect by the metalinguistic symbols X, $\mathscr{S}$, and $\mathscr{E}_E$, respectively, in the following.

As we have assumed at the beginning that L_c is a language of a classical (first-order) predicate logic, the standard classical semantics for this kind of logic holds true. Therefore, the logical symbols in $\mathscr{A}_c$ bear the standard logical interpretation and the binary relations of logical quasiorder $\subset$ and logical equivalence $\equiv$ are defined on Ψ_c. The quotient set $\Psi_c/\equiv$ can then be endowed with the partial order induced on it by $\subset$, which we still denote by $\subset$, and the poset $(\Psi_c/\equiv, \subset)$ is a Boolean lattice (the algebra of the propositions, or Lindenbaum–Tarski algebra, of L_c).

We endow the descriptive symbols in $\mathscr{A}_c$ with an effective interpretation, as follows. We preliminarily convene that bold symbols denote metalinguistic variables in the sequel. Then, we introduce a set I of *laboratories*, whose elements are interpreted as finite space-time domains which are physically equivalent, and to every $i \in I$ we associate a domain $\mathscr{D}_i$ whose elements are interpreted as *physical objects* (i.e., individual physical systems). Now, we assume that any *interpretation* σ of the individual variables maps every individual variable into a physical object in every laboratory $i \in I$. Furthermore, the symbols of state are interpreted, following Ludwig's (1983) interpretation, as nouns of classes whose elements are physically equivalent preparations; then, in every laboratory $i \in I$, every symbol of state $\mathrm{S} \in \mathscr{S}$ is associated to a subset $\rho_i(\mathrm{S})$ of $\mathscr{D}_i$, called *extension of* S in i, which is interpreted as the set of all physical objects that are actually prepared in i according to any preparation in the class denoted by the symbol S. Analogously, the symbols of exact effect are interpreted, again following Ludwig (1983), as nouns of classes whose elements are physically equivalent dichotomic registering devices, each of which exactly tests whether a given physical property can be attributed to the physical object that we want to examine; then, in every laboratory $i \in I$, every symbol of exact effect $\mathrm{E} \in \mathscr{E}_E$ is associated to a subset $\rho_i(\mathrm{E})$ of $\mathscr{D}_i$, called *extension of* E in i, which is interpreted as the set of physical objects in i that would give a positive answer if tested with any of the registering devices collected in the class denoted by the symbol E.

It should be carefully noted that the extension of a symbol of state is an *actual* set of physical objects according to the above definition; on the contrary, the extension of a symbol of exact effect is a *potential* set of physical objects. This difference, which characterizes our approach, has some relevant epistemological and technical consequences. Indeed, it implies that physical objects, on which measurements can be performed, are actually produced in every laboratory; yet their properties are considered theoretical expectations, not a real outcome of registering procedures, though actual measurements could be made, if desired. This avoids, in particular, any problem with "state changes induced by measurements." In addition, our definition of extension implies that the set of all extensions of symbols of state in a laboratory i is a partition on $\mathscr{D}_i$, so that the set of all preparations is not endowed with the structure type "selection procedure" as in the Ludwig approach.

5. TESTABILITY AND QUANTUM LOGIC

We have constructed and interpreted the classical language L_c. In order to fulfill our program, we are interested in singling out sets of formulas in L_c which correspond to physical statements that can actually be tested, if desired, that is, sets of testable formulas of L_c. In our framework, this is the same as saying that we are looking for sets of formulas whose truth value can be concretely determined by means of the registration devices introduced in the effective semantical interpretation of L_c.

Let us assume that an interpretation σ of the individual variables is assigned such that every $\mathbf{x} \in X$ is interpreted in every laboratory $i \in I$ on a given physical object. Then, it is apparent that at least the following two subsets of formulas are testable in the above sense.

(a) All the atomic formulas that can be obtained by substitution in the metalinguistic scheme of formulas $\mathbf{E}(\mathbf{x})$, where $\mathbf{E}$ ranges over the symbols of exact effects in $\mathscr{E}_E$ and $\mathbf{x}$ ranges over the individual variables in X. Indeed, any formula of this kind, say $E(x)$, can be interpreted as follows:

"the physical object denoted by x has the property tested by the exact effect denoted by E"

which obviously is a testable wff in every laboratory.

(b) All the molecular formulas that can be obtained by substitution in the metalinguistic scheme of formulas $(\forall \mathbf{x})(\mathbf{S}(\mathbf{x}) \rightarrow \mathbf{E}(\mathbf{x}))$, where $\mathbf{S}$ ranges over the symbols of states in $\mathscr{S}$, $\mathbf{E}$ ranges over the symbols of exact effects in $\mathscr{E}_E$, and $\mathbf{x}$ ranges over the individual variables in X. Indeed, any formula of this kind, say $(\forall x)(S(x) \rightarrow E(x))$, can be interpreted as follows:

"every physical object that is prepared according to the state denoted by S has the property tested by the exact effect denoted by E"

which is a testable wff in every laboratory, since the number of physical objects prepared according to a given state is necessarily finite, though it may be quite large.

It should be noted that the formulas in these two subsets are not testable in the same sense. In the former set, a single test is required in order to determine the truth value of a given statement, but simultaneous testability is generally prohibited. In the latter, a number of tests is required in order to obtain the truth value of any statement, but one can prove (Garola, 1991), under reasonable assumptions, that the truth values of different statements can always be determined simultaneously. This suggests that the two sets can be distinguished by means of suitable refinements of the concept of testability, but we do not insist on this point here, for brevity's sake.

It is also apparent that the above subsets can be immediately enlarged by noticing that every wff which is logically equivalent to a testable wff is also testable.

Let us assume now that the metalinguistic variables $\mathbf{x}$ and $\mathbf{S}$ are substituted by a specific individual variable, say x, and a specific symbol of state, say S, respectively, and let us denote the sets of wffs of Ψ_c which are obtained in this case from the metalinguistic schemes $\mathbf{E}(\mathbf{x})$ and $(\forall \mathbf{x})(\mathbf{S}(\mathbf{x}) \rightarrow \mathbf{E}(\mathbf{x}))$ by

Φ_c^x and Φ_c^S, respectively. Furthermore, let us denote by Ψ_c^x the set of wffs of Ψ_c which are logically equivalent to a formula at least in Φ_c^x, and by Ψ_c^S the set of wffs of Ψ_c which are logically equivalent to a formula at least of Φ_c^S (hence, trivially, $\Phi_c^x \subseteq \Psi_c^x$ and $\Phi_c^S \subseteq \Psi_c^S$).

The sets Ψ_c^x and Ψ_c^S are endowed with structure properties that are crucial in our approach since some of them can be interpreted as properties of the concept of testability in QP.

Let us begin with the logical structures of Ψ_c^x and Ψ_c^S. Each of these subsets is endowed with the quasiorder relation $\subset$ induced on it by restriction of the logical quasiorder $\subset$ defined on Ψ_c, and by a canonical "logical" equivalence relation $\equiv$ induced by $\subset$; furthermore, $\subset$ induces on $\Psi_c^x/\equiv$ and $\Psi_c^S/\equiv$ two order relations, which we still denote by $\subset$. Let us consider the posets $(\Psi_c^x/\equiv, \subset)$ and $(\Psi_c^S/\equiv, \subset)$. It is apparent that they are subposets of the Boolean lattice $(\Psi_c/\equiv, \subset)$; nevertheless, they are not necessarily sublattices of $(\Psi_c/\equiv, \subset)$ and, whenever they are lattices, they are not necessarily Boolean. It is also evident that no further information about $(\Psi_c^x/\equiv, \subset)$ and $(\Psi_c^S/\equiv, \subset)$ can be obtained by means of purely logical arguments.

Let us come to the empirical structures of Ψ_c^x and Ψ_c^S. Our investigation of these structures requires a preliminary examination of the semantical relations between the exact effects in the framework of a given physical theory. Now, it is easy to see that both in CP and in QP the set $\mathscr{E}_E$ can be endowed with an empirical partial order $\prec$ by setting

for every $\mathbf{E}_1, \mathbf{E}_2 \in \mathscr{E}_E$,

$$\mathbf{E}_1 \prec \mathbf{E}_2 \text{ iff for every laboratory } i,\ \rho_i(\mathbf{E}_1) \subseteq \rho_i(\mathbf{E}_2)$$

This partial order relation is endowed with properties that hold true in both theories; to be precise, a set of plausible physical assumptions can be made which imply that $(\mathscr{E}_E, \prec)$ is a complete orthocomplemented atomic lattice in both cases. Then, further assumptions can be introduced which differentiate CP from QP. However, the discussion of all these assumptions is lengthy and requires nontrivial technical tools. Since we aim at simplicity here, we choose a more direct way in order to endow $(\mathscr{E}_E, \prec)$ with a suitable structure. More precisely, standing on the effective interpretation of $\mathscr{E}_E$, we identify the poset $(\mathscr{E}_E, \prec)$, up to isomorphisms, with Piron's lattice of propositions, or Mackey's lattice of questions, or Ludwig's lattice of decision effects [indeed, these structures can be considered isomorphic; see Garola and Solombrino (1983)]. Then, $(\mathscr{E}_E, \prec)$ turns out to be a complete, orthocomplemented, atomic lattice, which is distributive in CP, orthomodular, and satisfies the covering law in QP.

Let us consider now Ψ_c^x and Ψ_c^S. By definition of Ψ_c^x, every wff $\mathbf{A}(x)\in\Psi_c^x$ is logically equivalent to an atomic wff $\mathbf{E}_A(x)\in\Phi_c^x$, with $\mathbf{E}_A\in\mathscr{E}_E$. Hence the mapping

$$\omega^x\colon\quad \mathbf{A}(x)\in\Psi_c^x\to\mathbf{E}_A\in\mathscr{E}_E$$

maps Ψ_c^x onto $\mathscr{E}_E$. Analogously, every $\mathbf{A}(S)\in\Psi_c^S$ is logically equivalent to a molecular wff $(\forall x)(S(x)\to\mathbf{E}^A(x))\in\Psi_c^S$, with $\mathbf{E}^A\in\mathscr{E}_E$. Hence the mapping

$$\omega^S\colon\quad \mathbf{A}(S)\in\Psi_c^S\to\mathbf{E}^A\in\mathscr{E}_E$$

maps Ψ_c^S onto $\mathscr{E}_E$. Consequently, the empirical partial order $\prec$ defined on $\mathscr{E}_E$ induces, via ω^x and ω^S, a quasiorder relation both on Ψ_c^x and Ψ_c^S, and a partial order relation both on $\Psi_c^x/\equiv$ and $\Psi_c^S/\equiv$ (we must remember that the symbol $\equiv$ denotes logical equivalence); in order to avoid the use of too many symbols, all these relations will still be denoted by $\prec$. It is then apparent that the posets $(\Psi_c^x/\equiv, \prec)$ and $(\Psi_c^S/\equiv, \prec)$ are order isomorphic to $(\mathscr{E}_E, \prec)$. Thus, we conclude that they also are complete orthocomplemented atomic lattices which are distributive in CP, orthomodular, and satisfying the covering law in QP.

This result is the basic one in our approach, and it is sufficient in affirming that we have attained our goals. Indeed, it shows that the lattices $(\Psi_c^x/\equiv, \prec)$ and $(\Psi_c^S/\equiv, \prec)$ are endowed with the mathematical properties required for QL in QP. Furthermore, because of the interpretation supplied for the formulas in Ψ_c^x and Ψ_c^S, the "propositions" which are elements of $\Psi_c^x/\equiv$, or $\Psi_c^S/\equiv$, can be endowed with two semantical interpretations, respectively, which are known possible interpretations of the propositions of QL. Thus, we can say that we have recovered quantum logical structures within a classical framework with extensional semantics.

Of course, the quasiorder relation $\prec$ on Ψ_c^x, or Ψ_c^S, is determined by empirical laws of physics (CP or QP) in our context; since Ψ_c^x and Ψ_c^S have been selected as subsets of testable formulas of L_c, we can say that the properties of this quasiorder relation in CP or QP formalize properties of the metalinguistic concept of testability within CP or QP. Thus, the mathematical properties of QL can be interpreted in terms of testability in QP, as desired (from this viewpoint, QL should be probably better classified as "pragmatics").

I would like to close this section by pointing out that I have not discussed the links between the logical order $\subset$ on $\Psi_c^x/\equiv$, or $\Psi_c^S/\equiv$, and the empirical order $\prec$ defined on the same subset. Since I have not explicitly exposed here the physical assumptions which justify the lattice structure of $(\mathscr{E}_E, \prec)$, I cannot formulate a detailed analysis of these links. Nevertheless, it is intuitively clear from the definition of $\prec$ that $\prec$ and $\subset$ coincide on

$\Psi_c^x/\equiv$. On the contrary, the order $\prec$ is actually stronger on $\Psi_c^S/\equiv$ than the order $\subset$.

6. CONCLUSIONS

Let us end with some further remarks which show the epistemological relevance of our results and indicate some possible applications.

(a) It follows from our discussion that QL is isomorphic to mathematical structures whose basic axioms have the truth mode of physical rather than logical laws if a suitable interpretation is given. According to our viewpoint, these structures are selected by the concept of testability, which is theory-dependent.

(b) QL is recovered in the framework of CL, which suggests that a nonstandard logic is not needed in QP.

(c) The basic classical language for QL may be endowed with extensional semantics, so that modal extensions of CL are not strictly needed and the introduction of possible worlds can be avoided.

(d) Two algebraic structures have been obtained, i.e., $(\Psi_c^x/\equiv, \prec)$ and $(\Psi_c^S/\equiv, \prec)$, which are isomorphic to QL in QP and are endowed with distinct interpretations. Thus, QL splits into different syntactical structures, each of which has a different interpretation.

(e) The statements in Ψ_c^x and Ψ_c^S, respectively, regard individual physical systems (referred to as "physical objects") and a class of individual physical systems (the physical objects that "are in the state denoted by S"). The clear *a priori* distinction on the syntactical level between these kinds of statements in our approach is relevant from an epistemological viewpoint not only because it establishes a one-to-one correspondence between syntax and semantics, but also because many antinomies in QP may derive by confusing the individual statements in Ψ_c^x and the class statements in Ψ_c^S.

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